Math 350 **Winter 2016**

Test #2

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Show your work and simplify your solutions. Where appropriate, your solutions should include definitions and references to theorems.

1.(a) Evaluate $\cosh \frac{\pi i}{2}$.

(b) Find all complex values z such that $\sin z = \frac{i}{2}$.

2.(a) State the Fundamental Theorem of Calculus for contours.

(b) Let γ be the part of a circle about the origin that joins the points $z = -1 + i$ to $z = 1 + i$ traversed with positive orientation. Compute $\int_\gamma \frac{1}{z}\, dz$.

3.(a) State Cauchy's Integral Theorem.

(b) State the Deformation Invariance Theorem.

(c) Prove that Cauchy Integral Theorem implies the Deformation Invariance Theorem.

4. Evaluate the following integrals:

(a) Let γ be the boundary of the square with vertices at $(\pm 1, \pm i)$ traversed once clockwise.

$$\oint_\gamma \frac{\sin z}{(4z + \pi)}\, dz$$

(b) Let $\gamma = \{\ z \in \mathbb{C} \ \big|\ |z - 2i| = 1\}$ with positive orientation.

$$\oint_\gamma \frac{z + i}{z^3 + 2z^2}\, dz$$

(c) Let γ be the circle of radius 2, centered at the origin, traversed once counterclockwise.

$$\oint_\gamma \frac{2 - z}{z^2 - z}\, dz$$

5. Let z_0 denote a fixed complex number, and let γ_R be a circle of radius R, centered at the point $z = z_0$, with positive orientation. Derive the following formula:

$$\oint_{\gamma_R} \frac{dz}{(z - z_0)^n} = \begin{cases} 0 & n \neq 1, \\ 2\pi i & n = 1. \end{cases}$$